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ON THE METHOD OF SOLUTION OF THE WAVE EQUATION WITH PERIODIC COEFFICIENTS

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ABSTRACT

The analytic method of solution of nonuniform wave equation of the problem of radiation of the magnetic moment, moving uniformly in the waveguide of arbitrary cross section with time – space periodic dielectric filling is given. The fields and the analytic expression for the Cerenkov energy losses of the magnetic moment in the region of strong (resonance) interaction between the radiation wave and the modulation wave are found.

Suppose that the source with magnetic moment $\vec{m}(0,0,m_z)$ moves uniformly along the axis (oz – axis) of an ideal waveguide of arbitrary cross-section with nonmagnetic filling, whose permittivity is modulated in space and time according the periodic law

$$\varepsilon = \varepsilon_0 [1 + m \cos(k_0 z - ut)], \quad (1)$$

where m is the modulation index, k_0 and u are the wave number and the phase velocity of the modulation wave, ε_0 is the permittivity of the waveguide filling in the absence of modulation.

It can be shown that the longitudinal component of the magnetic vector $H_z(x, y, z, t)$ as a potential of the transverse – electric (TE) field ($E_z = 0, H_z \neq 0$) satisfies the following nonuniform partial differential equation

$$\Delta_{\perp} H_z + \frac{\partial^2 H_z}{\partial z^2} - \frac{1}{c^2} \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial H_z}{\partial t} \right) = 4\pi \Delta_{\perp} m_z, \quad (2)$$

where $\Delta_{\perp} = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$, c is the velocity of the light in vacuum.

In (2) let us pass to the variables

$$\xi = z - ut, \eta = \frac{z}{u} - \frac{u}{c^2} \int_0^{\xi} \frac{d\xi}{1 - \beta^2 \frac{\varepsilon}{\varepsilon_0}}, \quad (3)$$

where $b = 1 - \beta^2$, $\beta = u / c \sqrt{\varepsilon_0}$. After any algebraic transformations we receive

$$\Delta_{\perp} H_z + \frac{\partial}{\partial \xi} \left[\left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_0} \right) \frac{\partial H_z}{\partial \xi} \right] - \frac{\varepsilon}{c^2 \left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_0} \right)} \frac{\partial^2 H_z}{\partial \eta^2} = -4\pi\varphi, \quad (4)$$

where

$$\varphi = -\Delta_{\perp} m_z, m_z = m_0 \frac{\delta(x-x_0)\delta(y-y_0)}{2\pi|u-v|} \int_{-\infty}^{\infty} e^{i\gamma(\eta-\eta_0)} d\gamma, \quad (5)$$

$$\eta_0 = \frac{v\xi}{u(v-u)} - \frac{1}{u} \int_0^{\xi} \frac{d\xi}{1 - \beta^2 \frac{\varepsilon}{\varepsilon_0}}, \quad (6)$$

(x_0, y_0) is the point of intersection of trajectory of the magnetic moment with the cross section of the waveguide, m_0 is the magnetic moment of the point source.

The equation (4) we can solve, suppose, that

$$H_z = \sum_{n=0}^{\infty} \psi_n(x, y) \int_{-\infty}^{\infty} e^{i\gamma\eta} H_n(\xi) d\gamma, \quad (7)$$

where $\psi_n(x, y)$ are the eigenfunctions of the second boundary value problem for the transverse section of the waveguide, and expanding the right part of equation (4) on eigenfunctions $\psi_n(x, y)$. Thus from (4) we receive the ordinary differential equation of second order

$$\frac{d}{d\xi} \left[\left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_0} \right) \frac{dH_n(\xi)}{d\xi} \right] + \frac{\chi_n^2}{1 - \beta^2 \frac{\varepsilon}{\varepsilon_0}} H_n(\xi) = -4\pi f_n(\xi), \quad (8)$$

where

$$f_n(\xi) = -\frac{m_0 \chi_n^2}{2\pi|u-v|} e^{i\gamma\eta_0} \psi_n(x_0, y_0), \quad (9)$$

$$\chi_n^2 = \frac{\gamma^2}{c^2} \varepsilon - \lambda_n^2 \left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_0} \right). \quad (10)$$

The equation (8) in the variable

$$S = \frac{k_0 b}{2\varepsilon_0} \int_b^\xi \frac{d\xi}{1 - \beta^2 \frac{\varepsilon}{\varepsilon_0}} \quad (11)$$

has a form

$$\frac{d^2 H_n(s)}{ds^2} + \frac{4\varepsilon_0^2 \chi_n^2}{k_0^2 b^2} H_n(s) = -4\pi \left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_0} \right) f_n(s), \quad (12)$$

where

$$f_n(s) = \frac{m_0 \lambda_n^2}{2\pi |u - v|} e^{-i \frac{2\gamma}{uk_0} \left(\frac{v}{v-u} - \frac{1}{b} \right) s} \psi_n(x_0, y_0). \quad (13)$$

The equation (12), as a Mathiew – Hill differential equation, we can solve, used the method, developed in our early articles [1–2]. Assuming a small modulation index m , after any transformations we receive the expression for H_z in variables z and t to the first approximation with respect to m . It gives the possibility to investigate the character of radiation in the region of strong interaction between the radiation wave and the modulation wave and find the Cerenkov energy losses of the magnetic moment in this case in the form

$$\left(\frac{dW}{dt} \right)_n = \frac{\pi m_0^2 \operatorname{sgn} \left(1 - \frac{uv}{c^2} \varepsilon_0 \right) \left(1 - \frac{v^2}{c^2} \varepsilon_0 \right) (1 - \beta^2)}{\varepsilon_0 (u - v) \left(1 - \frac{uv}{c^2} \varepsilon_0 \right)} \psi_n^2(x_0, y_0). \quad (14)$$

Note that the result for the case of stationary but nonuniform filling of the waveguide we can receive from (4) passing there to the limit when $u \rightarrow 0$.

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